

**Textbook:** *An Introduction to Manifolds* by Loring Tu

**Goal:** Understand examples of vector spaces, modules, and algebras that occur in the study of manifolds. We'll study the special case where our manifold is  $\mathbb{R}^n$ . These examples will give us the opportunity to understand how the objects studied in linear algebra are used in another area of mathematics.

**Course of Study:**

**2.1** Directional Derivative

**2.2** Germs of Functions

- Show that the definition of a *germ* is an equivalence relation.
- Write up problem 2.2

**2.3** Derivations at a Point

- Why is  $D_v : C_p^\infty \rightarrow \mathbb{R}$  an  $\mathbb{R}$ -linear map?
- Write up problem 2.3

**2.4** Vector Fields

- Write up the details of why the collection of smooth vector fields is a vector space.
- Why is  $\mathfrak{X}(U)$  a  $C^\infty(U)$ -module?

**2.5** Vector Fields as Derivations

- Write up problem 2.1
- **Optional:** Write up the discussion at the end of the section formally.